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Let $m=p^2-q^2$. $\therefore (y^2-x^2+1)z^2=(p^2-q^2)^2(p^4+q^4-4p^2q^2)$.

Let $p^4+q^4-4p^2q^2=(p^2-2rq^2)^2$. $\therefore p^2=\dfrac{4r^2-1}{r-1}.\dfrac{q^2}{4}$.

When $r=13$, $p^2=\frac{225}{16}q^2$, $m=\frac{209}{16}q^2$.

$\therefore xz=\frac{209}{256}(241)q^4$, $yz=\frac{209}{256}(269)q^4$, $z=\frac{209}{256}(149)q^4$.

Deleting the common factor $\frac{209}{256}$, $xz=241q^4$, $yz=269q^4$, $z=149q^4$, $(x^2+y^2-1)z^2=(329)^2q^8$, $(x^2-y^2+1)z^2=(89)^2q^8$, $(y^2-x^2+1)z^2=(191)^2q^8$.

Let $q=1$, and the least numbers all different are 241, 260, 149.

By giving q different values an infinite number of solutions can be found.

AVERAGE AND PROBABILITY.

137. Proposed by G. H. HARVILL, Malakoff, Texas.

A, *B*, *C*, and *D*, playing whist, agree that the person who first cuts an ace shall have a stake of \$313. What is the value of each person's expectation before the play begins, each taking his turn at cutting in the order named as the game progresses?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., J. E. SANDERS, Hackney, Ohio, and J. SCHEFFER, A. M., Hagerstown, Md.

The chance that A cuts an ace$=\frac{1}{13}$, that he does not$=\frac{12}{13}$.
The chance that B cuts an ace$=\frac{1}{13}(\frac{12}{13})$, that he does not$=(\frac{12}{13})^2$.
The chance that C cuts an ace$=\frac{1}{13}(\frac{12}{13})^2$, that he does not$=(\frac{12}{13})^3$.
The chance that D cuts an ace$=\frac{1}{13}(\frac{12}{13})^3$, that he does not$=(\frac{12}{13})^4$; etc.

$\therefore A$'s chance$=\frac{1}{13}[1+(\frac{12}{13})^4+(\frac{12}{13})^8+(\frac{12}{13})^{12}+\ldots\ldots]=\frac{2197}{7825}$.
B's chance$=\frac{1}{13}(\frac{12}{13})[1+(\frac{12}{13})^4+(\frac{12}{13})^8+(\frac{12}{13})^{12}+\ldots\ldots]=\frac{2028}{7825}$.
C's chance$=\frac{1}{13}(\frac{12}{13})^2[1+(\frac{12}{13})^4+(\frac{12}{13})^8+(\frac{12}{13})^{12}+\ldots\ldots]=\frac{1872}{7825}$.
D's chance$=\frac{1}{13}(\frac{12}{13})^3[1+(\frac{12}{13})^4+(\frac{12}{13})^8+(\frac{12}{13})^{12}+\ldots\ldots]=\frac{1728}{7825}$.
A's expectation$=\frac{2197}{7825}$ of \$313$=\$87\frac{6886}{7825}$;
B's expectation$=\frac{2028}{7825}$ of \$313$=\$81\frac{939}{7825}$;
C's expectation$=\frac{1872}{7825}$ of \$313$=\$74\frac{6886}{7825}$;
D's expectation$=\frac{1728}{7825}$ of \$313$=\$69\frac{939}{7825}$.

138. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

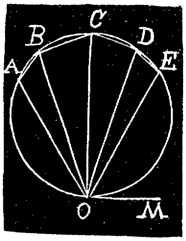
Find the average area of, (1) triangle, (2) quadrilateral, (3) pentagon, (4) hexagon, formed by taking, (1) three, (2) four, (3) five, (4) six random points on the circumference of a given circle radius a.

Solution by J. E. SANDERS, Hackney, Ohio, and the PROPOSER.

We will solve for hexagon first. Let $\angle AOM=\theta$, $\angle BOM=\phi$, $\angle COM=\psi$, $\angle DOM=\rho$, $\angle EOM=\delta$.

Then $AO=2a\sin\theta$, $BO=2a\sin\phi$, $CO=2a\sin\psi$, $DO=2a\sin\rho$, $EO=2a\sin\delta$, where a=radius of circle.

Area of hexagon$=u=2a^2[\sin\theta\ \sin\phi\ \sin(\theta-\phi)+\sin\phi\ \sin\psi\ \sin(\phi-\psi)+\sin\psi\ \sin\rho\ \sin(\psi-\rho)+\sin\rho\ \sin\delta\ \sin(\rho-\delta)]$.



$$\therefore \Delta = \frac{\int_0^\pi \int_0^\theta \int_0^\phi \int_0^\psi \int_0^\rho u d\theta\, d\phi\, d\psi\, d\rho\, d\delta}{\int_0^\pi \int_0^\theta \int_0^\phi \int_0^\psi \int_0^\rho d\theta\, d\phi\, d\psi\, d\rho\, d\delta}$$

$$= \frac{120}{\pi^5} \int_0^\pi \int_0^\theta \int_0^\phi \int_0^\psi \int_0^\rho u d\theta\, d\phi\, d\psi\, d\rho\, d\delta$$

$$= \frac{120a^2}{\pi^5} \int_0^\pi \int_0^\theta \int_0^\phi \int_0^\psi [2\rho \sin\theta \sin\phi \sin(\theta-\phi)$$

$$+2\rho \sin\phi \sin\psi \sin(\phi-\psi) + 2\rho \sin\psi \sin\rho \sin(\psi-\rho)$$

$$+\sin^2\rho - \rho\sin\rho\cos\rho] d\theta\, d\phi\, d\psi\, d\rho \ldots\ldots (1),$$

$$= \frac{30a^2}{\pi^5} \int_0^\pi \int_0^\theta \int_0^\phi [4\psi^2 \sin\theta \sin\psi \sin(\theta-\phi) + 4\psi^2 \sin\phi \sin\psi \sin(\phi-\psi) + 3\psi$$

$$-3\sin\psi \cos\psi - 2\psi^2 \sin\psi \cos\psi] d\theta\, d\phi\, d\psi \ldots\ldots (2),$$

$$= \frac{20a^2}{\pi^5} \int_0^\pi \int_0^\theta [2\phi^3 \sin\theta \sin\phi \sin(\theta-\phi) + 3\phi^2 - \phi^3 \sin\phi \cos\phi - 3\sin^2\phi] d\theta\, d\phi \ldots\ldots (3),$$

$$= \frac{5a^2}{2\pi^5} \int_0^\pi [10\theta^3 - 15\theta - 2\theta^4 \sin\theta \cos\theta + 15\sin\theta \cos\theta] d\theta = \frac{15a^2}{2\pi^5}\left(1 - \frac{3}{\pi^2}\right).$$

For the pentagon the points can be taken $\frac{1}{5}$ the number of ways for a hexagon. Writing θ for ϕ in the three last terms of (3), we get for the pentagon,

$$\Delta = \frac{4a^2}{\pi^4} \int_0^\pi [3\theta^2 - \theta^3 \sin\theta \cos\theta - 3\sin^2\theta] d\theta = \frac{5a^2}{\pi^4}\left(1 - \frac{3}{2\pi^2}\right).$$

For the quadrilateral, the points can be taken $\frac{1}{20}$ the number of ways for a hexagon. Writing θ for ψ in the last three terms of (2), we get,

$$\Delta = \frac{3a^2}{2\pi^3} \int_0^\pi [3\theta - 3\sin\theta \cos\theta - 2\theta^2 \sin\theta \cos\theta] d\theta = \frac{3a^2}{\pi}.$$

For the triangle the points can be taken $\frac{1}{60}$ the number of ways for a hexagon. Writing θ for ρ in the last two terms of (1), we get

$$\Delta = \frac{2a^2}{\pi^2} \int_0^\pi [\sin^2\theta - \theta\sin\theta \cos\theta] d\theta = \frac{3a^2}{2\pi}.$$

These last three follow at once from the fact that for the pentagon, quadrilateral, and triangle, we have three, two, and one term in the expression for the area, while the power of π is for the pentagon π^4, for the quadrilateral π^3, for the triangle π^2, in the denominator.

139. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.

Remark by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

No. 139 is the same as No. 130 for which I have sent a solution previously.

140. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Obtain the average area of a triangle formed by a tangent to the four—cusped hypocycloid and the coördinate axes.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let a=radius of fixed circle. Then portion of tangent intercepted by coördinate axes=a. Area of triangle=$\frac{1}{2}xy$, subject to the condition $x^2+y^2=a^2$.

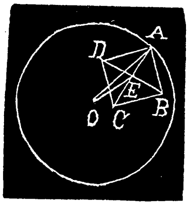
$$\therefore \text{Average area}=\tfrac{1}{2}\int_0^a x\sqrt{(a^2-x^2)}dx \Big/ \int_0^a dx=\tfrac{1}{6}a^2.$$

141. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Upon a circular table, radius r, a *variable* square plate is thrown at random. What is the probability that the plate will lie wholly on the table?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let O be the center of the given circle, radius r; $ABCD$ the square, center E; $AB=2x$, $OE=z$, $\angle OAE=\theta$. Then $AE=x\sqrt{2}$.



If the center of the square, E, falls on a circle center O and radius $(r-x\sqrt{2})$, the square will be wholly on the table. If E falls on a circle, center O and radius z, the plate will lie wholly on the table. $z=\sqrt{[r^2+2x^2-2rx\sqrt{(2)}\cos\theta]}$. The limits of x are 0 and $\frac{1}{2}r\sqrt{a}$; of θ, 0 and $\frac{1}{4}\pi-\sin^{-1}(x/r)=\theta'$. Let p=chance. Since the whole number of ways E can fall on the circle is πr^2, we get,

$$p=\frac{\pi\int_0^{\frac{1}{2}r\sqrt{2}}[r-x\sqrt{2}]^2dx}{\pi r^2\int_0^{\frac{1}{2}r\sqrt{2}}dx}+\frac{\pi\int_0^{\frac{1}{2}r\sqrt{2}}\int_0^{\theta}z^2dx\,d\theta}{\pi r^2\int_0^{\frac{1}{2}r\sqrt{2}}\int_0^{\theta}dx\,d\theta}$$